PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 1 JULY 15, 1915 Number 7

NOVA GEMINORUM NO. 2 AS A WOLF-RAYET STAR

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Presented to the Academy, June 7, 1915

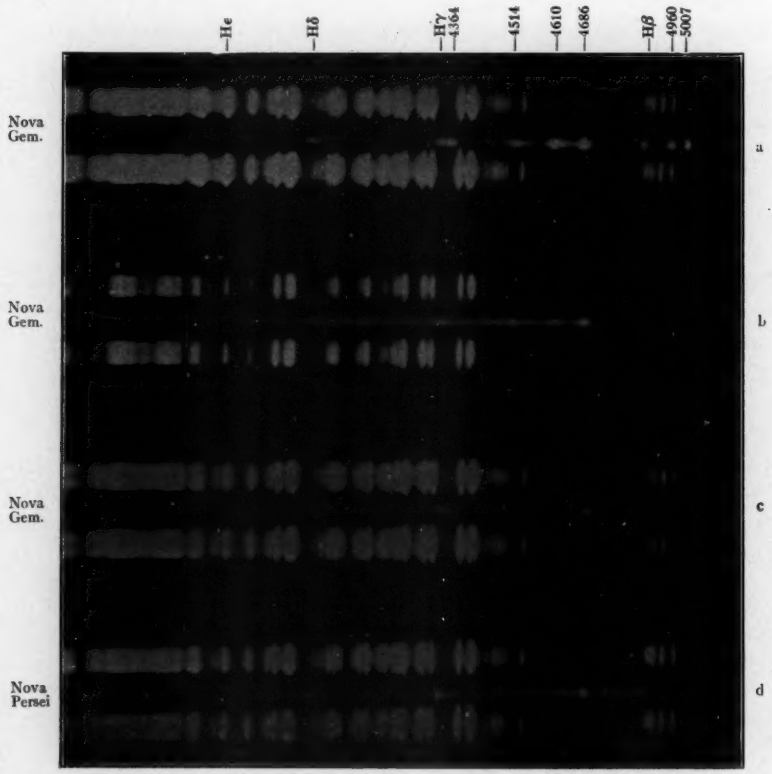
It has been known for many years that the spectrum of temporary stars or Novae develops in a later stage into a spectrum nearly identical with that of planetary nebulae. First observed in the case of Nova Aurigae of 1891, the same result has been found to hold for all of the temporary stars observed since that time, including the most important Nova of recent years, Nova Persei of 1901. The question whether this nebular spectrum may be regarded as the final stage in the development of the spectra of temporary stars is of great importance because of its immediate bearing on the order of evolution of stars. In his book on Stellar Movements Eddington says in this connection:[1] "There is some justice in a remark of R. T. A. Innes: 'The fact that we have seen a star change into a nebula ought to outweigh every contrary speculation that stars originate from nebulae.' "

The faintness of the Novae in the later stages of their history has made observations of their spectra exceptionally difficult. In 1906, however, Hartmann[2] obtained an observation of Nova Persei, then of about the eleventh magnitude, which showed that two of the principal nebular lines had disappeared, and he called attention to the strong similarity of its spectrum to that of one of the Wolf-Rayet stars. Since that time the work has been taken up at Mount Wilson, with the 60-inch reflector and extended to include observations of other Novae.

Photographs of the spectrum of Nova Geminorum No. 2 (1912) taken during the past year have shown some peculiarly interesting developments in this most recent of the temporary stars. In February 1914, the spectrum consisted of prominent bright bands joined by a continuous spectrum of rather feeble intensity. These bands are

those so conspicuous in the spectra of nebulae, the more important in order of brightness being λ 4686, 5007, 4610, 4364, 4960 and the hydrogen lines, $H\beta$, $H\gamma$, $H\delta$ and $H\epsilon$.

During the summer the star could not be observed on account of its position, but in November another photograph was secured. This showed some extraordinary changes. In addition to a great increase



in the intensity of the continuous spectrum, the plate showed the disappearance of the chief nebular line at λ 5007 and the line so commonly associated with it at λ 4364, and a marked reduction in the intensity of λ 4610. The absence of λ 5007 is especially remarkable in view of the presence, although in reduced intensity, of the nebular line at λ 4960. These two lines, known as N_1 and N_2 of the nebular spectrum, without exception so far as we know, have always been observed together, and vary in intensity together, λ 5007 being about twice as strong as λ 4960.

Although no common origin has been established for these lines this peculiarity in behavior might be compared to the disappearance of D_2 in the sodium spectrum, while D_1 persisted in reduced intensity.

Another photograph of the spectrum of the star was obtained in April 1915. This showed that all of the characteristic nebular lines had disappeared, the line λ 4686 which is common to the spectra both of nebulae and Wolf-Rayet stars, and the hydrogen lines alone remaining. There are, however, traces of some of the Wolf-Rayet bands and the continuous spectrum is very strong.

In a communication published by us in 1914[3] we compared the spectrum as then observed of Nova Geminorum No. 2 with that of three of the temporary stars discovered in previous years. Two of these, Nova Aurigae (1891) and Nova Persei (1901), were found to have strong continuous spectra on which the hydrogen lines appeared as bright bands of moderate intensity. No characteristic nebular lines were present in their spectra, but λ 4686 was very prominent in Nova Persei, and present, though much less bright, in Nova Aurigae. The close agreement of these spectra and especially that of Nova Persei with the spectrum of some of the Wolf-Rayet stars was referred to in the course of the comparison. The interesting result now appears that the spectrum of Nova Geminorum No. 2 has also developed into that of a Wolf-Rayet star, being essentially identical with that of Nova Persei, although the hydrogen lines are somewhat weaker. Measurements of the negatives of Nova Persei and Nova Geminorum show the presence of the following Wolf-Rayet bands in addition to λ 4686 and the hydrogen lines:

Nova Persei	4021	4200	448	4514	454	461	4634
Nova Geminorum	402	4199	444	4512		461	469

In Plate I are shown direct enlargements of three spectra of Nova Geminorum No. 2 and one spectrum of Nova Persei. The enlargement is about 17 fold. The dates of the photographs are as follows:

	Date	*Exposure time*
Nova Geminorum (a)	1914, Feb. 22	2 hours
Nova Geminorum (b)	1914, Nov. 16	2 "
Nova Geminorum (c)	1915, Apr. 10–12	8 "
Nova Persei (d)	1913, Nov. 27	8 "

The photograph of April 10–12 was taken under very poor observing conditions.

The especial feature of interest attaching to these results is the fact that a continuous series of observations in the case of Nova Geminorum No. 2 has shown the development of the spectrum of this star through

the successive stages characteristic of Novae into one very strongly resembling that of planetary nebulae; and then, by the gradual elimination of the nebular lines and their replacement by Wolf-Rayet bands, into a spectrum identical with this characteristic type of stellar spectra. It appears very probable from the observations of Nova Aurigae and Nova Persei that this stage of development is to be considered as in the nature of a permanent one. It is of interest to note, however, that the period required to reach this condition may be quite variable for different stars, since the nebular lines λ 5007 and λ 4960 were still by far the most prominent feature of the spectrum of Nova Lacertae (1910) as recently as the autumn of 1913. It seems entirely possible that the nebular lines found in the case of Novae are extraneous to the stars themselves and due to material in which they are involved. Of especial importance in this connection is the recent discovery by Wright[4] that the central star in certain planetary nebulae has the Wolf-Rayet type of spectrum.

[1] *Stellar Movements and the Structure of the Universe*, p. 156.
[2] *Astr. Nachr. Kiel*, **177**, 113 (1908).
[3] *Contrib. Mt. Wilson Solar Obs.*, No. **87**; *Astrophys. J.*, **40**, 294 (1914).
[4] *Astrophys. J.*, **40**, 466 (1914).

A SINGULAR DARK MARKING ON THE SKY

By E. E. Barnard

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Read before the Academy, April 20, 1915. Received, June 4, 1915

In the search for dark or vacant regions in the sky I have photographed a number of very remarkable places, such as those near *ρ Ophiuchi* and *θ Ophiuchi*, and elsewhere.

These regions are generally connected with the presence of diffused nebulosity, but I have found several cases in which a well defined dark object seems superposed on the sky itself without any visible presence of ordinary nebulosity. One of these, and perhaps the most remarkable on account of its well defined character and singular form, is in the position: (1855.0)$\alpha = 20$ h. 48 m.; $\delta = +59°.6$, in Cepheus (see Plate). It resembles an irregular, roughly torn and curved opening through which a darker region is visible. It is one degree long and narrows down to something like 3′ across at its western end. It seems to be a part of the background of the sky independent of the starry stratum (there are no indications whatever of nebulosity immediately about it), and is appar-

ently of a similar nature to the remarkable dark 'lanes' shown in my photographs of a region in *Taurus*. (See *Astrophys. J.*, **25**, 218, 1907). There are two very small dark spots near and south of the west end of the dark marking.

The accompanying plate is from a photograph which was taken by me on October 1, 1910, with the 10-inch Bruce telescope of the Yerkes Observatory, with an exposure of 6 h. 2 m. I had previously taken a photograph of this part of the sky on September 30, 1910, with an exposure of 5 h. 5 m. which verifies the above in all particulars.

NORTH

DARK OBJECT IN *CEPHEUS* ($\alpha = 20^h\ 48^m$, $\delta = +60°$)
Scale: 1°=41mm.

There are two possible explanations of this object:

(1) That it is an opening in a widely diffused nebulous stratum.

(2) That it is an opaque, non-luminous object projected against space, which is itself luminous.

From this object and those in *Taurus*, I get the impression that the interstellar spaces (or possibly the regions beyond the stars), perhaps covering the entire heavens, are suffused with a feeble nebulosity that, with very prolonged exposures, affects the photographic plate; and that such phenomena (as the present one) are due to the projection upon this background of nearer, dark, opaque objects. If not this, then they

are rifts or openings in a luminous background which look out into the blackness of space beyond. From investigations elsewhere in the sky, I lean towards the idea that these are relatively non-luminous, opaque bodies, seen against a luminous background.

THE RULING AND PERFORMANCE OF A TEN-INCH DIFFRACTION GRATING

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Presented to the Academy, May 13, 1915

The principal element in the efficiency of any spectroscopic appliance is its resolving power—that is, the power to separate spectral lines. The limit of resolution is the ratio of the smallest difference of wave-length just discernable to the mean wave-length of the pair or group. If a prism can just separate or resolve the double yellow line of sodium its limit of resolution will be (5896–5890)/5893 or approximately one one-thousandth, and the resolving power is called one thousand.

Until Fraunhofer (1821) showed that light could be analysed into its constituent colors by diffraction gratings these analyses were effected by prisms the resolving power of which has been gradually increased to about thirty thousand. This limit was equaled if not surpassed by the excellent gratings of Rutherford of New York, ruled by a diamond point on speculum metal, with something like 20,000 lines, with spacing of 500 to 1000 lines to the millimeter. These were superseded by the superb gratings of Rowland with something over one hundred thousand lines, and with a resolving power of 150,000.

The theoretical resolving power of a grating is given as was first shown by Lord Rayleigh by the formula $R = mn$, in which n is the total number of lines, and m the order of the spectrum. An equivalent expression is furnished by $R = \frac{l}{\lambda}(\sin i + \sin \theta)$, where l is the total length of the ruled surface, λ the wave-length of the light, i the angle of incidence, and θ the angle of diffraction; and the maximum resolving power which a grating can have is that corresponding to i and θ each equal to 90° which gives $R = 2l/\lambda$. that is twice the number of light waves in the entire length of the ruled surface.

This shows that neither the closeness of the rulings nor their total number determine this theoretical limit, and emphasizes the importance of a large ruled space.

This theoretical limit can be reached, however, only on the condition of an extraordinary degree of accuracy in the spacing of the lines. Several methods for securing this degree of accuracy have been attempted but none has proved as effective as the screw. This must be of uniform pitch throughout and the periodic errors must be extremely small.

For a short screw for example, one sufficient for a grating two inches in length, the problem is not very difficult but as the length of the screw increases the difficulty increases in much more rapid proportion. It was solved by Rowland in something over two years.

Since this time many problems have arisen which demand a higher resolving power than even these gratings could furnish. Among these is the resolution of doubles and groups of lines whose complexity was unsuspected until revealed by the interferometer and amply verified by subsequent observations by the echelon and other methods.

Others that may be mentioned in this connection are the study of the distribution of intensities within the spectral 'lines'; their broadening and displacement with temperature and pressure; the effect of magnetic and electric fields, and the measurement of motions in the line of sight, as revealed by corresponding displacement of the spectral lines in consequence of the Doppler effect.

All of these have been attacked with considerable success by observations with the echelon, the interferometer and the plane-parallel plate. These methods have a very high resolving power, but labor under the serious disadvantage that adjacent succeeding spectra overlap making it difficult to interpret the results with certainty.

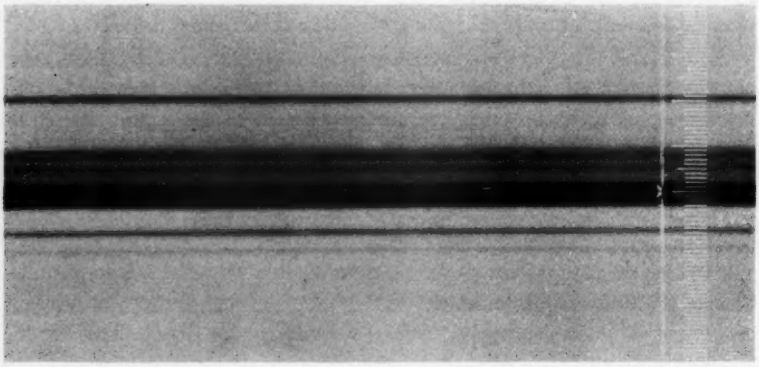
Some twelve years ago the construction of a ruling engine was undertaken with the hope of ruling gratings of fourteen inches—for which a screw of something over twenty inches is necessary. This screw was cut in a specially corrected lathe so that the original errors were not very large, and these were reduced by long attrition with very fine material until it was judged that the residual errors were sufficiently small to be automatically corrected during the process of ruling.

The principal claim to novelty of treatment of the problem lies in the application of interference method to the measurement and correction of these residual errors.

For this purpose one of the interferometer mirrors is fixed to the grating carriage, while a standard, consisting of two mirrors at a fixed distance apart, is attached to an auxiliary carriage. When the adjustment is correct for the front surface of the standard, interference fringes appear. The grating carriage is now moved through the length of the standard (one-tenth of a millimeter if the periodic error is to be

investigated; ten or more millimeters if the error of run is to be determined) when the interference fringes appear on the rear surface. This operation is repeated, the difference from exact coincidence of the central (achromatic) fringe with a fiducial mark being measured at each step in tenths of a fringe (twentieths of a light-wave). As a whole fringe corresponds to one hundred-thousandth of an inch, the measurement is correct to within a millionth of an inch.

The corresponding correction for periodic errors is transferred to the worm wheel which turns the screw; and for errors of run to the nut which moves the carriage. In this way the final errors have been almost completely eliminated and the resulting gratings have very nearly realized their theoretical efficiency.



Enlargement of photograph of the green mercury 'line' λ5461 taken by H. E. Lemon with 10-inch diffraction grating in sixth order. Scale: 1 division = 0.01A.U. Ruled surface, $9\frac{3}{8}$ x $2\frac{1}{4}$ inches, 11700 lines per inch. Mounted in Littrow form with 8-inch lens by Brashear. Focal length, 20 feet.

A number of minor points may be mentioned which have contributed to the success of the undertaking.

(*a*) The ways which guide the grating carriage as well as those which control the motion of the ruling diamond must be very true; and these were straightened by application of an auto-collimating device which made the deviation from a straight line less than a second of arc.

(*b*) The friction of the grating carriage on the ways was diminished to about one-tenth of that due to the weight (which may amount to twenty to forty pounds) by floating on mercury.

(*c*) The longitudinal motion of the screw was prevented by allowing its spherically rounded end to rest against an optically plane surface of diamond which could be adjusted normal to the axis of the screw.

(*d*) The screw was turned by a worm wheel (instead of pawl and ratchet) which permits a simple and effective correction of the periodic errors of the screw throughout its whole length.

(*e*) A correcting device which eliminates periodic errors of higher orders.

(*f*) It may be added that the nut which actuates the carriage had bearing surfaces of soft metal (tin) instead of wood, as in preceding machines. It was not found necessary to unclamp the nut in bringing it back to the starting point.

Finally it may be noted that instead of attempting to eliminate the errors of the screw—by long continued grinding which inevitably leads to a rounding of the threads—it has been the main object to make these errors conveniently small; but especially to make them constant—for on this constancy depends the possibility of automatic correction.

The accompanying photograph made with a 10-inch grating sixth order, (actual ruled surface 9.4 inches by 28 inches), used in the Littrow form with an excellent 8-inch lens by Brashear, is given in evidence of its performance. The resolving power as shown by the accompanying scale of angstrom units is about 450,000. The original negative shows a resolving power of about 600,000. The theoretical value is about 660,000.

Doubtless the possibility of ruling a perfect grating by means of the light-waves of a homogeneous source has occurred to many—and indeed this was one of the methods first attempted.

It may still prove entirely feasible—and is still held in reserve if serious difficulty is encountered in an attempt now in progress to produce gratings of twenty inches or more. Such a method may be made partly or perhaps completely automatic, and would be independent of screws or other instrumental appliances.

It may be pointed out that an even simpler and more direct application of light-waves from a homogeneous source is theoretically possible and perhaps experimentally realizable.

If a point source of such radiations sends its light-waves to a collimating lens and the resulting plane waves are reflected at normal incidence from a plane surface, stationary waves will be set up as in the Lippman plates; these will impress an inclined photographic plate with parallel lines as in the experiment of Wiener; and the only limit to the resolving power of the resulting grating is that which depends on the degree of homogeneity of the light used. As some of the constituents of the radiations of mercury have been shown to be capable of

interfering with difference of path of over a million waves, such a grating would have a resolving power exceeding a million.

This investigation has had assistance from the Bache Fund of the National Academy of Science, from the Carnegie Institution, and from the University of Chicago. In addition to the grateful acknowledgement to these institutions I would add my high appreciation of the faithful services rendered by Messrs. Julius Pearson and Fred Pearson.

A HIGHLY SENSITIVE ELECTROMETER

By A. L. Parson

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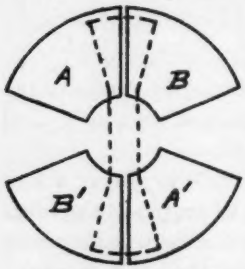
Presented to the Academy, June 3, 1915

In this instrument the principle of working in a condition approaching instability is used to increase greatly the sensitiveness of the quadrant electrometer, which, even in its ordinary form, is more sensitive than other electrometers in which this condition of instability has been employed. The box-shaped quadrants are replaced by flat sectors subtending about 70° each at the centre and arranged as shown in the figure. Because of the two large gaps between A and B′, and between A′ and B, the needle hangs stably, as shown by the dotted lines, by virtue of electric forces quite apart from the torsion of the suspension. (Incidentally, this makes it possible to use a torsionless suspension or a float for the needle, which latter has been tried with some success.) If now a potential-difference is set up between AA′ and BB′ the needle is deflected, at first nearly proportionately to the potential difference, and then less and less as the large gaps are approached.



A HIGHLY SENSITIVE ELECTROMETER

The approach to instability and hence an increase in sensitiveness is brought about by means of a mechanical device, which on turning a micrometer screw can be made to widen the small gaps between A and B and between A′ and B′, thus making the needle less and less stable in its central symmetrical position. In this way the sensitivity which in the least sensitive condition is about 3 mm. per millivolt at a scale distance of 5 metres can be raised to as much as 60 mm. per millivolt easily, and with some care even to 150 mm. per millivolt.

A most important feature of the instrument is that, on account of the relative unimportance of the thickness of the quartz suspension, the

sensitivity to potential-differences is practically independent of the size of the instrument. The last model made, with a needle 1.8 cm. long, has a capacity of 9 cm., but this could without difficulty be halved by reducing the size of the instrument; and it might be halved again by omitting one end of the needle and the pair of sectors below it.

Although the sensitivity of the electrometer (with the very efficient optical system used) is theoretically great enough to detect 10^{-6} volt, it has not as yet been made steady enough to detect an isolated potential-difference of less than about 3×10^{-5} volt.

The details of the construction of the instrument and the results of measurements made with it, together with a consideration of the cause of the residual unsteadiness will be discussed in a more extended article to be published shortly.

THE DISTRIBUTION AND FUNCTIONS OF TRIBAL SOCIETIES AMONG THE PLAINS INDIANS: A PRELIMINARY REPORT

By Clark Wissler

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Presented to the Academy, June 2, 1915

For several years the American Museum of Natural History has been engaged in systematic anthropological field-work among the surviving Indians of the great Plains area in North America pertaining to one definite problem, viz., the distribution and functions of tribal societies. Early observers noted the existence of societies, chiefly for men, which within the tribe seemed to be correlated and in some cases organized into progressive series, or ranks. Some data more or less fragmentary were recorded by Lewis and Clark, Catlin, and Maximilian in early days, and later by Grinnell and J. O. Dorsey. The first serious investigation of such societies was undertaken by A. L. Kroeber among the Arapaho. This was followed by work among the Cheyenne by G. A. Dorsey, among the Assiniboin by R. H. Lowie, and among the Blackfoot by me. These pioneer studies revealed such striking similarities between these four tribal systems of societies as to suggest a case of culture diffusion.

The significance of the problem may appear from the following brief statement; two investigators had previously made special use of what data were then available on these societies in their respective efforts to explain such phenomena as manifestations of a yet to be discovered law of social evolution.[1] In each case the method was the same, one employed by many ethnologists and sociologists; viz., to collect examples of tribal societies from several parts of the world and to theoretically

correlate them to form a consistent development series. The chief weakness of this method lies in the inadequacy of the data; for, so far as I know, the successive tribal organizations for an entire geographical area have never before been the object of detailed investigation. Hence, it is clear that such discussions as we have cited can at best be but hypothetical since the data upon which they are based do not in any way reveal the true characters of the phenomena involved.

Our investigation was planned to face the other way, or to proceed by observation and the collection of concrete data upon all the tribes of a geographical area as a preliminary condition to the interpretation of the phenomena.

The plan of work was developed in 1906 and was to include special investigations among the following tribes: Arikara, Blackfoot, Comanche, Crow, Dakota, Hidatsa, Iowa, Kansas, Kiowa-Apache, Mandan, Pawnee, Plains-Cree, Plains-Ojibway, Ponca, Sarsi, Shoshone, and Ute. The societies of the Arapaho, Gros Ventre, Omaha, Osage, Kiowa, and Cheyenne were excepted because they either had been or were then under investigation by others.

It is clear that an investigation of such magnitude could not be conducted by a single individual but must proceed by the coöperation of several field-workers. Accordingly, the problem was taken up as a departmental investigation conducted by the writer and his associates in the American Museum of Natural History, Dr. P. E. Goddard, Dr. R. H. Lowie, and Mr. Alanson Skinner. The field-work has been completed and the reports upon the several tribes published preparatory to an exhaustive correlation of the data, which will form the final paper to the volume.[2]

Some of the most important results of this final study may be summarized as follows:

1. It is now clear that the distinction of age found in some tribes, or where only persons of certain ages were eligible to membership, has no functional relation to the organization in which it is found for the very simple reason that in many cases what is obviously the same organization is found among other tribes independent of an age system. (Schurtz's conclusion was that all these societies were natural developments from simple age groups.)

2. The one feature that is almost universal is the exercising of police functions by some one or all of the tribal series of societies, but that the societies are expressions of a social tendency toward control is unlikely because they are often but the secondary or deputy police called upon for special service by permanent officials. In most cases the society is

called only for policing a buffalo hunt and rarely for ordinary services. Again, we have the policing of the buffalo hunt among some tribes by appointed individuals and so independent of a society.

3. Among some tribes these societies are predominately military and based upon military ideals while in others they have serious ceremonial and religious characters. This is well illustrated in the kit-fox society, one of the most widely distributed which among the Teton-Dakota is controlled by military ideals but among the Blackfoot is one of their most sacred organizations.

4. If a general résumé of the characteristics of societies for the different tribes is made, we find their ideals of organization or conceived functions exceedingly variable, much more so than the objective forms of organization, for there is marked uniformity in the number and duties of officers, kinds of regalia, and even in the names of individual societies. It is therefore difficult to consistently interpret these organizations as expressions of any special function in the tribal life, but rather as due to many and various causes.

While the data make it clear that by culture diffusion these societies have been spread from tribe to tribe, it has been diffusion of a desultory kind since in no case has a system of societies been carried along but only separate organizations. We have just noted how the same society appeared in different associations among different tribes. Further, a statistical study of the details of organization reveals a similar condition for it often happens that what is a distinctive feature of a society in one tribe will be found associated with quite a different organization in another. From this it appears that the phenomena can be most readily explained as due to mutual tribal borrowings. We have succeeded in tracing certain features of organization to particular tribes, so it is now clear that no one tribe can be the originator of the society system as a whole.

Whatever may be the ultimate interpretation of the data our work shows clearly that such ethnic phenomena can be made the object of scientific investigation and that very fundamental social problems can be successfully approached by proper inductive methods.

[1] H. Schurtz, *Altersklassen and Männerbunde, eine Darstellung der Grundformen der Gesellschaft*, Berlin, 1902; H. Webster, *Primitive Secret Societies*, New York, 1908.

[2] *Anthropological Papers*, American Museum of Natural History, volume 9. The descriptive papers are on the Teton-Dakota, Blackfoot, and Pawnee by Clark Wissler; the Eastern Dakota, Crow, Hidatsa-Mandan, Arikara, and Shoshonean tribes by Robert H. Lowie; the Plains-Cree, Plains-Ojibway, Iowa, Kansas, and Ponca by Alanson Skinner; the Sarsi and Kiowa-Apache by Pliny E. Goddard; and the final discussions will be contributed by Lowie and Wissler.

THE DETERMINATION OF SURFACE-TENSION

By T. W. Richards and L. B. Coombs
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Received, May 20, 1915

This investigation is part of a series of investigations having for their object the study of the fundamental properties of liquids. It is hoped that, when a number of these properties have been determined with great accuracy, the essential relations between them may be discovered with greater certainty than is possible at present.

Among the significant properties of liquids surface-tension stands out as one of the most interesting. This somewhat unfortunately named property affords us a valuable clue concerning the cohesive forces which bind the substance together and cause it to become a liquid. Therefore, its exact determination is a matter of far-reaching importance to anyone who seeks to understand the fundamental nature of the liquid state.

A glance at the published data concerning surface-tension leads one to conclude that much remains to be done. For example, the values obtained by experienced men for water at 20° vary all the way from 70.6 to 78, according to different methods. Even a single method (for example, that of the rise in capillary tubes) has yielded results in the last twenty-five years varying from 70.6 to 72.7, and no satisfactory evidence is forthcoming as to the reasons for the difference.

It seemed therefore worth while not only to study the surface-tension of a variety of new liquids, but also to discover the reason for the divergences between different methods, and to obtain results of absolute as well as of relative accuracy for liquids already studied. The present work, although only preliminary, seems to have been successful in locating several of the heretofore not adequately heeded sources of error.

The method chosen was the well known method of capillary rise in carefully measured tubes, because this method seems to be one of the most direct and least likely to lead to insoluble mathematical complications. The method has been used by many experimenters in the past.

Several features of the present work deserve emphasis. In the first place, the careful selection of the capillary tubes and the calibration by means of short columns of mercury received especial attention. Correction was made for the meniscuses of these columns in determining the exact diameter of the tube. Again, great care was taken to determine exactly the position of the meniscuses, both of the larger and of the smaller surface, by means of a finely adjustable black screen behind the tubes to be measured. It was found that the exact position was

only to be observed when the meniscus appeared to be precisely tangent to the edge of the screen.

Both of these precautions have been more or less fully heeded by others; but another precaution, the determination of the diameter necessary for the larger tube in order to secure perfectly flat surface, has been often overlooked. We found that a tube over 35 mm. in diameter was required, and that even into this wide tube it was not permissible to insert a capillary; for such an insertion acted as another basis of support for the liquid and caused appreciable rise. By actual measurement we found that the capillary rise in a 20 mm. tube, counting only from the middle of the bottom of the meniscus, was over 0.5 mm., and the addition of a capillary tube in the middle of this raised it at least 0.3 mm. more. As apparatus of this sort has been used by most experimenters on surface-tension, most of the capillary rises which have been reported are in the neighborhod of 1 mm. too low—an error which accounts for a large part of the discrepancies between different methods.

APPARATUS IN DIAGRAMMATIC SECTION. (THE LOADED SINKER IS TO DIMINISH THE NECESSARY VOLUME OF LIQUID.)

Another error which does not seem to have received sufficient attention is that due to the weight of the liquid in the finer meniscus above its lowest point. The equation of Poisson, which is usually used for calculating this weight, gives an absurd result with tubes as wide as 1 cm., and therefore must be rejected. Another equation, that of Desains, gives a result for fine tubes which is not plausible; hence this also seemed unworthy of confidence. A careful measure of the height of the meniscus between its lowest point and its line of contact with the fully wetted walls showed that in very fine tubes this height is almost exactly equal to the radius, and that, therefore, the meniscus is here essentially hemispherical. As the tube widens, the hemisphere becomes somewhat flattened, and for moderate radii it appears entirely safe to apply, as a correction to be added on account of the meniscus, one-third of the meniscus height as actually measured. This method of correction was shown to give consistent results with tubes of different diameters.

Great difficulty and considerable liability for error were found in the inequalities of the glass of the tubes to be measured. Accordingly all measurements were made in reversible apparatus of the type shown in the diagram. This form of apparatus when exactly half filled with liquid is observed, first in an upright position in front and behind, and again in an inverted position in front and behind. Thus from the average, all the displacements due to refraction of irregular walls of the larger tube are entirely eliminated. The regularity of the walls of the smaller tube are tested by the calibration in the first place.

The preliminary results thus obtained are recorded in the table given below. All the measurements were made in the presence of air. The surface-tensions are calculated according to the well known equation $\gamma = \frac{1}{2} rhg\,(s_1 - s_2)$ in which the angle of contact of the meniscus in the tube is assumed to be zero.

CAPILLARY CONSTANTS AND SURFACE TENSIONS

Data obtained with Apparatus III (20.00°)

Radius of capillary = 1.0099 mm.

SUBSTANCE	AVERAGE HEIGHT IN MM.	CORRECTION FOR SMALL MENISCUS	CORRECTED HEIGHT IN MM.	DENSITY 20°/4°	CAPILLARY CONSTANT a^2	SURFACE TENSION DYNES PER MM²
Water	14.394	0.321	14.715	0.99823	14.861	72.62
Benzene	6.351	0.311	6.662	0.8788	6.728	28.94
Toluene	6.369	0.311	6.680	0.8658	6.736	28.58

Data obtained with Apparatus IV (20.00°)

Radius of capillary = 0.1936 mm.

Benzene	34.620	0.061	34.681	0.8788	6.714	28.89
Methyl alcohol	30.063	0.061	30.124	0.7918	5.832	22.61
Ethyl alcohol	29.720	0.061	29.781	0.7892	5.766	22.27
Isobutyl alcohol	30.016	0.061	30.077	0.8019	5.823	22.86
Ethyl butyrate	29.403	0.061	29.464	0.8789	5.704	24.53

It will be noted that, in general, these results are higher than most of the earlier results, for the reasons already suggested. For example, Quincke found only 14.47 as the capillary constant of water, and Renard and Guye found 6.47 for that of benzene. The carefully obtained results of Walden and Swinne, although measured in a fairly satisfactory apparatus, are all subject to the same error, because the apparatus was calibrated by means of a value of the capillary constant of benzene which is too low. It is hoped that an exact evaluation of the amount of this error may make it possible to correct the comprehensive and valu-

able work of Ramsay and Shields and Aston, as well as that of Renard and Guye, to the standard herewith shown to be more trustworthy.

This paper is only a preliminary communication. A fuller report of the work will appear in the July number of the *Journal of the American Chemical Society*. Much more work upon the subject has already been finished, and yet more is in prospect. We hope that yet further accuracy may be attained in the future, bearing in mind the precautions to which attention has been called in this paper, and that the results may be capable of fruitful discussion.

In conclusion, we are glad to express our indebtedness to the Carnegie Institution of Washington for some of the apparatus we have employed.

Summary.—In the course of a series of determinations of capillary constants by measuring the capillary rise in fine tubes, the following precautions have been especially emphasized: (1) The detection and correction of inequalities in the glass tubes employed were effected by the use of a reversible apparatus. (2) Reference of the capillary rise was made to an unrestricted flat surface 38 mm. in diameter, the largest ever used. It was shown that much smaller surfaces are too small and that the insertion of a capillary in the middle of a larger tube causes appreciable error by increasing the capillary effect of the large tube. (3) Especial care was taken that the true bottom of the meniscus should be read. (4) The weight of the fine meniscus was in each case allowed for, and a new approximate formula was suggested for its calculation, depending upon the observed height of the meniscus.

Heeding these precautions, determinations of the capillary constants of several important liquids were determined at 20° as follows: water 14.861, benzene 6.721, toluene 6.736, methyl alcohol 5.832, ethyl alcohol 5.793, isobutyl alcohol 5.823, ethyl butyrate 5.704.

AN EXHIBIT IN PHYSICAL ANTHROPOLOGY

By Aleš Hrdlička

DIVISION OF PHYSICAL ANTHROPOLOGY, U. S. NATIONAL MUSEUM, WASHINGTON

Presented to the Academy, June 8, 1915

In the April number of the PROCEEINGS OF THE NATIONAL ACADEMY OF SCIENCES, I published a brief account of "Some recent anthropological explorations," which were carried out under my direction or by myself, for the Smithsonian Institution and the Panama-California Exposition at San Diego, in different parts of the world. I shall now point, in an equally brief way, to the material results of these expeditions in relation with the Exposition.

One of the main objects of the whole work was to bring together a comprehensive, instructive and harmonious exhibit relating to the natural history of man. Such exhibits have been attempted with more or less success at a number of former expositions, from that in Paris (1878) to those in Chicago (1892) and Dresden (1911); but all these efforts, while reaching creditable and useful results, suffered from the inclusion of archeological and other exhibits, as well as from defects due to heterogeny or lack of material, and especially to the impossibility of carrying out a uniform and necessarily costly plan.

In undertaking the preparation of the exhibits for San Diego, the whole subject of Anthropology or Man's Natural History, was divided into four sections. These were (1) Man's Evolution (Phylogeny); (2) Man's Development and Growth (Ontogeny); (3) Man's Variation; and (4) Man's Decline and Elimination. To these it seemed necessary to add a subdivision dealing with the modern means by which anthropology works and including a comprehensive library. In accordance with this plan, a stipulation was made for five connected moderate sized halls, to accommodate in logical progression the five sections. The exhibits were to be, furthermore, as far as possible original and of permanent museum value, so that they could serve if circumstances permitted as a nucleus or an important part of a future permanent anthropological museum on the Pacific Coast.

As mentioned in the former paper, only three years were available for the carrying out of the above large program. Illness among the staff, the European war, and other conditions interfered. Not a few of the links or specimens needed were not to be found or purchased; and other difficulties developed. In consequence, the results are not as complete and perfect as they might be; yet it is safe to say that in richness, instructiveness, and harmony, they surpass considerably anything attempted before in this line. These facts seem to justify the present remarks which call attention not so much to the existence of the collection at San Diego as to the material progress accomplished in a branch of science in which exhibits are particularly scarce and difficult.

The five moderate sized halls which harbor the exhibits occupy a special building marked 'Science of Man.'

The first hall is devoted entirely to Man's Evolution. It contains one of the largest existing series of accurate, first-hand casts of the most important authentic skeletal remains of early man. Added to this are numerous attempted reconstructions of the generally defective early crania. On the walls above the specimens are illustrations showing, in good sized photographic enlargements or colored charts, the locali-

ties of discovery of the various originals represented by the casts. Besides this, there are large charts relating to geology and stratiography so far as these relate to man; and charts showing the probable lines of man's ascent after the foremost authorities. Another series of illustrations, covering more than one entire wall, is devoted to the pictorial representations of early man, by the early man himself (cave drawings and sculptures), and by noted scientists and artists of the present day. The whole center of this hall is occupied by the most striking and interesting series of busts—reconstructions of early man, made by the talented Mascré and under the direction of R. Rutot, one of the foremost European students of early man. Finally, there is a large series of original specimens showing in a progressive way the crania of existing primates, or more exactly those from the lemur to man. The anthropoid apes are each represented in this series by skulls of a full grown male and female and by one of a young animal of the same species.

The second hall, or that devoted to *Ontogeny*, contains six series of true-to-nature busts, made at the National Museum by one of the best modelers in this country, and showing, by different age stages, from birth onward and in both sexes, the three principal races of this country, namely the 'thorough-bred' white American (at least three generations American on each parental side), the Indian, and the full-blood American negro. These series form a unique, costly exhibit, nothing like which has ever been attempted before in this or any other country. Each set consists of fifteen busts and proceeds from infants at or within a few days after birth, to the oldest persons that could be found. The oldest American negro represented is 114. After the new-born, the stages are 9 months, 3 years, 6, 10, 15, 20, 28, 35, 45, 55, 65, and 75 years, and then the oldest person obtainable. Special care was exercised in ascertaining the age of the subjects, particularly among the Negroes and the Indians. No choice was made of the subject beyond that due to the requirements of pedigree, age, and good health. The whites and negroes were obtained with a few exceptions in the District of Columbia and vicinity, but their places of birth range over a large part of the eastern, southern, and middle states; for the Indian I chose the Sioux, a large, characteristic, and in a very large measure still pure blood tribe, and one in which the determination of the ages in the subjects was quite feasible.

On the walls of this room are eight original charts relating to the physical decline of man or normal senility, and a series of other charts, mostly in colors, relating to the development of the child in different races. In a case along the walls are series of brains, skulls and other

bones of the body, almost all in original specimens, showing the development of these parts from early fetal to adult life, or even towards old age.

The third hall is devoted to racial, sexual, and individual variations. The main exhibits consist of ten pairs of original busts, showing important groups of humanity; and over one hundred original bronzed masks, illustrating the individual variation in physiognomy among different races. The latter series includes the masks of 19 Bushmen, which are especially rare and valuable. The wall is covered with charts showing racial classification, distribution, and statistics. The middle of the hall is given to specially constructed steel fixtures, for over two hundred colored and plain portrait transparencies of racial types. These portraits include several special series, such as 'The Indian child in different parts of the continent,' 'Beauty among Indians' etc. In the wall case are numerous exhibits of original specimens relating to racial, sexual and individual variation in the skull and other parts of the skeleton.

The fourth hall is devoted to the illustration of the causes which, outside of strictly normal senility, contribute to the decline of the human organism, and in the vast majority of cases cause death. The geographical distribution of the principal diseases is represented in a series of small colored maps on the wall, and there are charts relating to causes and frequency of mortality. The main exhibit in this room, however, consists of a very large series of skeletal remains of the prehistoric American Indian, showing his entire, or almost entire pathology. This exhibit is unique and could scarcely be duplicated. It is supplemented by a large number of Indian crania which show wounds and by 60 original cases of pre-Columbian Indian trephining for such wounds.

The fifth hall is fitted out as a modern anthropological laboratory and is also used as a lecture room in which frequent demonstrations are given bearing on the collections in this section. There are, further, the library; two cases of modern anthropometric instruments; anthropometric charts on the walls, and a series of portraits of the most eminent deceased representatives of anthropology.

It will be seen from the above that the exhibits here briefly described, are quite different from the usual exhibits at an Exposition. They are of permanent value, have attracted from the first most encouraging attention, and are capable of forming the foundation of an anthropological center in a locality eminently fitted for such a purpose.

THE COMPRESSIBILITIES OF THE ELEMENTS AND THEIR RELATIONS TO OTHER PROPERTIES

By T. W. Richards

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Presented to the Academy, May 22, 1915

Eight years ago the compressibilities of thirty-five elements were determined at Harvard, in order to discover the relation of these compressibilities to one another and to other properties of these fundamental substances. All the values were referred to that of mercury, with the understanding that, although the compressibility of mercury was not at that time very well known, they could all be easily corrected to the true standard when more complete knowledge of this quantity had been attained.

This time now seems to have come. The recent research of Bridgman at high pressures and that of the author in conjunction with E. P. Bartlett at lower pressures have furnished such a consistent and convincing outcome that the compressibility of mercury is no longer in serious doubt. Accordingly, it is appropriate that the older results should be corrected to the true standard. Moreover, a better standard of pressure is now available, and the pressure gauges used in the early work have been very carefully compared with it, so that the older results can be corrected in this respect also.

The table below contains the corrected values of the compressibilities in question, together with new values for tungsten and tantalum, determined with the help of E. P. Bartlett, and of boron, determined with the help of J. H. Hodges.

The megabar (or megabarie) is chosen as the measure of pressure because this absolute unit bids fair to become the scientific standard of the future, being the pressure of one megadyne per square centimeter. It is almost 2% greater than the kilogram per square centimeter, and 1.3% less than the 'atmosphere.'

In addition to the compressibilities, the table records also several other properties, which will be shown to be more or less related. Many of these also have been determined at Harvard, but some are due to the work of others. Most of these latter are easily found in the literature; complete references would occupy far too much space for the present publication.

Compressibilities, Atomic Volumes, Densities, Melting Points, Coefficients of Expansion and Atomic Weights of a Majority of the Commonly Solid and Liquid Elements Between 100 and 500 Megabars (if the compressibility of mercury = 0.00000395).

	Average Compressibility at 20° (see above)	*Atomic Volume*	*Density at 20°*	*Melting Point (absolute temp.)*	*Cubic Coefficient of Expansion × 100,000*	*International Atomic Weight (1914)*
Lithium	9.0	13.1	0.534	453	19.	6.94
Boron	0.3	4.7	2.34	2800		11.0
Diamond	[0.7] (?)	3.4	3.51	Very high	0.3	12.00
Graphite	3.	5.4	2.26	Very high	7.2	12.00
Sodium	15.6	23.7	0.971	371	22.	23.00
Magnesium	2.9	13.3	1.74	927	7.8	24.32
Aluminium	1.47	10.1	2.60	930	7.2	27.1
Silicon	0.32	11.4	2.42	1733	2.3	28.3
Phosphorus, red	9.2	14.4	2.15	863		31.04
Phosphorus, white	20.5	16.6	1.82	317	36.	31.04
Sulphur	12.9	15.5	2.07	384	18.	32.07
Chlorine	[95] (?)	25.	1.42	171		35.46
Potassium	31.7	45.5	0.862	335	25.	39.10
Calcium	5.7	25.3	1.53	1073		40.07
Chromium	0.9	7.7	7.06	1823		52.0
Manganese	0.84	7.7	7.37	1533		54.93
Iron	0.60	7.1	7.85	1790	3.6	55.84
Nickel	0.43	6.7	8.7	1725	4.2	58.68
Copper	0.75	7.1	8.92	1356	5.0	63.57
Zinc	1.7	9.5	7.13	692	8.7 (?)	65.37
Arsenic	4.5	13.3	7.53	1073	1.6	74.96
Selenium	12.0	18.5	4.28	490	13.	79.2
Bromine	52.	25.1	3.12	266		79.92
Rubidium	40.	56.0	1.53	312	[30] (?)	85.45
Molybdenum	0.46	11.1	10.23	2773	1.1	96.0
Palladium	0.54	9.3	12.13	1822	3.8	106.7
Silver	1.01	10.3	10.5	1134	5.7	107.88
Cadmium	2.1	13.0	8.60	594	7.4	112.40
Tin	1.9	16.2	7.29	505	6.7	119.0
Antimony	2.4	17.9	6.71	903	3.3	120.2
Iodine	13.	25.7	4.94	386	25.0	126.92
Cesium	61.	71.0	1.87	301	[33] (?)	132.81
Tantalum	0.53	10.9	16.67	3123	2.3	181.5
Tungsten	0.27	9.6	19.23	3300(?)	1.1	184.0
Platinum	0.38	9.1	21.31	2020	2.7	195.2
Gold	0.66	10.2	19.24	1336	4.3	197.2
Mercury	3.95	14.8	13.55	234		200.6
Thallium	2.3	17.2	11.83	574	9.0	204.0
Lead	2.33	18.2	11.37	600	8.8	207.2
Bismuth	3.0	21.2	9.80	543	4.0	208.0

In the few cases where the compressibilities have been determined by others, especially by Grüneisen and Bridgman, the agreement with the results given above is good; and there is every reason to believe that these figures represent closely the true values of the constants in question. Therefore, they are worthy of further study, especially as regards their mutual relations and their relations to other properties.

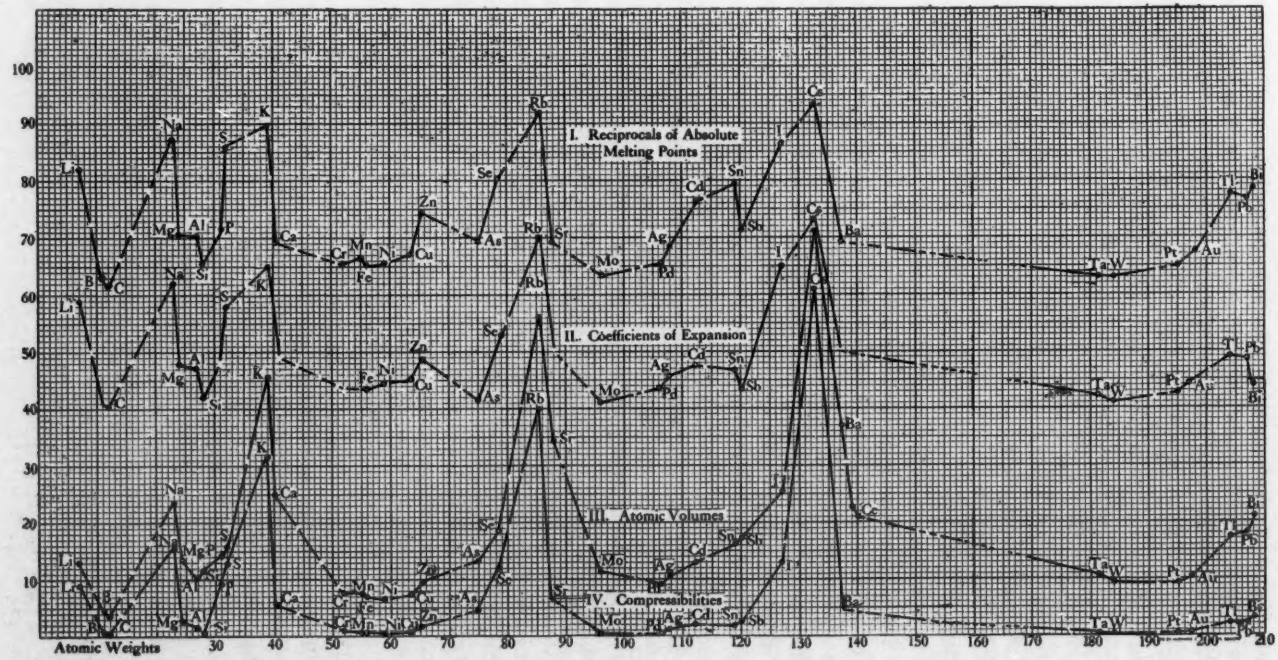

Diagram showing: I. Reciprocals of Absolute Melting Points; II. Coefficients of Expansion; III. Atomic Volumes; IV. Compressibilities of a Majority of the Commonly Solid and Liquid Elements between 100 and 500 Megabars.

Leaving out of consideration the behavior of the liquids, which doubtless is not fairly comparable with that of solids, we may compare these various quantities in parallel curves, each being plotted in relation to the increasing atomic weight. The accompanying diagram presents in the lowest line compressibilities; in the next above, atomic volumes; in the curve no. II, coefficients of expansion, and in the highest curve, the reciprocals of the absolute melting points.

That essential relations exist between these data cannot be doubted. In each case a maximum is seen to occur with the alkali metals; and although the rest of the curves are less similar, nevertheless, distinct connection is traceable. The compressibility curve is most like the atomic volume curve, and the coefficient of expansion curve most like that of the reciprocals of the melting points.

The attempt to find a mathematical relationship between these quantities is an interesting one. Such a relationship has already been sought by Grüneisen, Einstein, Debye, and others, upon the basis of a speculative theory, without much experimental support. With the wealth of data herewith presented, however, an empirical attempt is possible. It is soon evident on studying the figures that in cases where the melting points and atomic volumes are nearly similar, as, for example, in the cases of gold and silver, the denser substance is the less compressible. On the other hand, with similar density and melting point, as in the case of sodium or potassium, the substance with the greater atomic volume is the more compressible. Yet again, where the atomic volumes and densities are not very different, as in the case of sodium and calcium, the substance with the higher melting point has the lower compressibility. Taking account of these general tendencies, one sees that the equation should take this general form:

$$\text{Compressibility} = K\frac{f_1(V)}{f_2(D)f_3(T_m)}$$

in which K represents a constant, V the atomic volume, D the density, T_m the absolute melting point, f_1, f_2, f_3, various as yet undetermined functions.

The search for the nature of these respective functions is tedious and has not been carried very far; but the substitution in this equation of $K = 0.00021$, $f_1(V) = V$, $f_2(D) = D^{0.25}$ and $f_3(T_m) = (T_m - 50)$ gave results of about the right order with nearly all of the substances. For example, lithium is thus calculated to be 8 instead of 9, sodium, 15.6 (exactly its observed value), copper 0.66 instead of 0.75, silver 1.02 instead of 1.01, tungsten 0.29 instead of 0.27, etc. Some of the cal-

culated values, to be sure, deviated much more widely than this, but none is of entirely a different order from the truth. It is possible to show a necessary relationship between this empirical equation and the outcome of recent work by Grüneisen, and there is also some relationship between this equation and the well-known formula of Dupré, but the full discussion of these relationships would occupy more space than is available in the present publication. It may be noted, however, that the equation of Dupré is less satisfactory than the empirical equation just given as regards the elements, for density plays too large a rôle in the older expression. Moreover, the equation of Grüneisen is designed only for presumably monatomic elements, and it gives a negative (and therefore absurd) result for bismuth, showing that its range is more limited than that of the equation just mentioned.

No pretense is made, of course, that the empirical equation herewith communicated is a final statement of the relation of the quantities concerned. Nevertheless, because it gives the approximate order of magnitude of the compressibilities of widely different substances, one has a right to believe that the basis of the equation really refers to some of the more important tendencies which are at work.

Attention may be called to the fact that these relationships are in accord with those which would be expected from the prediction of the theory of compressible atoms. An atom much distended because of its internal makeup might be supposed to be highly compressible, and so might one much distended because not greatly pressed upon by outside cohesive pressure. Both of these tendencies would cause small density, and, accordingly, large atomic volume, and the second of these tendencies would add low melting point. Therefore, the general form of the equation is plausible.

A more complete discussion of the data and equation will soon be given in the *Journal of the American Chemical Society*.

Summary.—This paper records all the recent work on the compressibility of the elements performed at Harvard, reduced to the best available standard—the newly determined compressibility of mercury. It is pointed out that the reciprocals of the melting points are very closely associated with the coefficients of expansion, and that both of these properties seem to be essentially connected with atomic volume and compressibility. A preliminary empirical equation is proposed, which shows roughly the nature of this relationship.

RADIAL VELOCITIES WITHIN THE GREAT NEBULA OF ORION

By Edwin B. Frost

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Presented to the Academy, June 9, 1915

The *Astrophysical Journal* for October, 1914, contained an important paper by MM. Bourget, Fabry, and Buisson of Marseille, summarizing the results of their application to the Orion nebula of the photographic interferometric methods of M. Fabry. They found variations in the radial velocity from one point to another within the nebula, differences as large as 10 km. per second at points quite close to each other being indicated by local deformations of the interference rings. They also detected great collective movements, the northeastern region receding, and the southwestern region approaching, relatively to the mean velocity at the trapezium, with velocities of about 5 km. per second.

It seemed of interest to repeat these determinations independently by the standard spectrographic method, and toward the close of the winter observations were begun at Yerkes Observatory with the Bruce spectrograph, arranged with a dispersion of one prism. The radial velocity was inferred from the displacements of the hydrogen lines β and γ and the lines of nebulium at λ 5007 and 4959. At the position of the western star of the trapezium, No. 619 in Bond's catalogue of stars in the nebula, the radial velocity of the nebula was found from measures of eight plates (taken occasionally during the past eleven years) to be $+15.6 \pm 0.5$ km. per second (recession). This is the mean of independent measures of each plate by the writer and Mr. C. A. Maney, and it agrees almost exactly with the value of $+15.8$ km. found by the observers at Marseille for the region of the trapezium.

For several other positions around the trapezium, and not over 2′ from it, we obtained values ranging from $+6$ to $+17$ km. The probable error for the velocity at one point, from the mean of measures by the two observers of a single plate, should be about ± 1.5 km., so that the reality of the difference may be regarded as fully confirmed. Exposures of from two to three hours were required at some of the positions, so that the accumulation of plates was slow, and additional observations will be required when the 'open season' for Orion returns.

We must accordingly alter our conceptions of the nebula as an enormous mass of quiescent gas, and regard it as seething with local whirlpools besides perhaps having a considerable motion of rotation as a whole.

THE RADIAL VELOCITIES OF THE MORE DISTANT STARS

By Walter S. Adams

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Presented to the Academy, June 8, 1915

As a result of the work of Kapteyn, Campbell, and Boss it has been recognized for some years that the linear motions of the brighter stars vary with their spectral types, the velocities of the solar type stars being higher than those of types B and A. It is equally well-known, however, that the solar type stars in general have larger proper motions and so are much nearer to the sun than those of earlier types. Hence the stars, for which the velocities have been compared, have quite different distances, and if, as was once suggested by Eddington,[1] the nearer stars move more rapidly than the distant ones this fact would account, in part at least, for the apparent variation of velocity with spectral type.

In the course of an analysis of the radial velocity results from the Lick Observatory and Mount Wilson, J. C. Kapteyn derived the relationship between radial velocity and proper motion for the K type stars,[2] and found that the velocity increased rapidly with the amount of proper motion. In this computation the effect of stream motion was eliminated to a large extent by a selection of stars nearly 90° from the vertices of the streams. The Mount Wilson observations of the spectra of stars having both small and large proper motions provide the material for a similar comparison for other types of spectra. This is given in Table I: the effect of stream motion, however, has not been eliminated. The average velocity v' is corrected for a solar motion of 20 km. directed toward the apex

$$\alpha = 17\,\mathrm{h}\,59\,\mathrm{m}. \quad \delta = +30°.8$$

and no velocities exceeding 100 km. are included.

TABLE I

SPECTRAL TYPE	NUMBER OF STARS	μ	v'	NUMBER OF STARS	μ	v'
B	61	0".016	8.2	52	0".041	9.6
A	55	0 .019	10.0	104	0 .067	10.7
F	20	0 .011	10.1	45	0 .530	24.6
G	63	0 .013	10.6	69	0 .670	24.9
M	27	0 .015	12.6	12	0 .170	17.6
To these we may add Kapteyn's value for the K stars with stream motion eliminated:						
K	27	0.013	10.9	19	>0.30	26.7

It is clear from these results that the radial velocity increases rapidly with proper motion. Only a few A type stars of very large proper motion are known. Of those observed at Mount Wilson for which μ exceeds 0″.20, two have velocities of over 150 km.; one has a velocity of 87 km.; and the average for the other six is 20 km.

In view of this relationship between proper motion and radial velocity it becomes of especial interest to compare the Mount Wilson observations of stars of small proper motion with the results of W. W. Campbell for stars selected on the basis of apparent magnitude alone. Table II contains the average values of the radial velocities corrected for solar motion, of 1034 stars published in Campbells' first table.[3] No constant K has been applied to these results. The proper motions have been taken from Boss's catalogue for the individual stars appearing in Campbell's later lists, and hence the mean values are not strictly correct for the stars whose velocities are given. Since the number of stars is large, however, it does not seem probable that these values can be seriously in error.

TABLE II

SPECTRAL TYPE	CAMPBELL				MOUNT WILSON		
	No. Stars for v'	No. Stars for μ	μ	v'	No. Stars	μ	v'
O and B....	141	224	0″.031	8.99	61	0″.016	8.23
A.....	133	206	0 .094	9.94	55	0 .019	10.04
F.....	159	192	0 .234	13.90	20	0 .011	10.14
G, K.....	529	549	0 .202	15.15	63	0 .013	10.60
					56	0 .014	11.53
M.....	72	78	0 .074	16.55	27	0 .015	12.56

The omission of 5 stars of the A type would reduce the proper motion for Campbell's stars from 0″.094 to 0″.079. The principal feature of this comparison is the relatively close agreement of the Lick and Mount Wilson results for the B and A stars, and the large difference for those of other types. This difference appears to be associated directly with the great increase in proper motion between the A and F type stars in Campbell's list. With the aid of the relationship already referred to between proper motion and radial velocity deduced by Kapteyn for the K stars it is possible to apply corrections to Campbell's results to reduce to the proper motion 0″.031 of the B stars. The resulting values are shown under v' in Table III. In addition both the Lick and the Mount Wilson velocities have been corrected for the effect of stream motion by the approximate method devised by Eddington.[4]

TABLE III

SPECTRAL TYPE	CAMPBELL			MOUNT WILSON		
	v'	Corrected[1]	μ	v'	Corrected[1]	μ
O and B.....	9.0	9.0	0".031	8.2	8.2	0".016
A.....	8.6	6.8	0 .031	10.0	7.7	0 .019
F.....	9.0	7.8	0 .031	10.1	8.8	0 .011
G, K	11.0	9.6	0 .031	10.6 11.5	9.2 10.0	0 .013 0 .014
M.....	15.6	13.6	0 .031	12.6	10.9	0 .015

[1] v' corrected for stream motion.

In his later discussion of the solar motion as derived from the several spectral types Campbell has given the average radial velocities with a constant correction K applied to the velocity of each star. This constant has a value ranging from about zero for the F and G stars to over 4 km. for those of type B. If these results are treated in the same way as those of Table III we obtain the following values:

TABLE IV

TYPE	v'	CORRECTED	μ
O and B.....	6.5	6.5	0".031
A.....	9.6	7.4	0 .031
F.....	9.5	8.3	0 .031
G.....	9.1	7.9	0 .031
K.....	13.2	11.5	0 .031
M.....	16.1	14.0	0 .031

The constant K as used by Campbell is the average velocity v' taken according to sign for the stars of each spectral class, and is dependent upon the value of the solar motion. For all of the Mount Wilson stars the same value, $V = 20$ km., has been used. This gives the following average velocities taken according to sign:

B +1.26; A −0.24; F −0.86; G +0.05; K −1.18; M +0.31.

A change in the value of V from 20 km. to 19 km. would reduce the value for the B stars from +1.26 km. to +1.06 km. These quantities must be regarded as very moderate in size.

It appears clear from the Mount Wilson results given in Table III that the variation in velocity with spectral type is very gradual for these distant stars; and except in the case of the M stars, which, because of their number, are of comparatively low weight, the same conclusion may be drawn from Campbell's values after allowance has been made for the effect of the large number of relatively near stars included among his

F, G, and K type spectra. This would be in agreement with the hypothesis of Eddington, already referred to, that the relation between velocity and spectral type might be a relation between velocity and distance, the stars nearest the sun, mainly the types F, G, and K, moving more rapidly than the distant stars. Eddington considered this hypothesis as disproved because an analysis of the A type stars indicated no increase of radial velocity with increasing proper motion. The fact that such an increase exists in the case of the later type stars, however, is shown clearly in Table I. Because of the slight range in proper motion a similar variation for the B and A stars is less certain, although indicated on the face of the results.

The principal feature of interest in this comparison of proper motion and radial velocity is the low average velocity found for the distant stars of types F to M. These stars are on the average stars of high absolute luminosity, and the possibility of a relationship between radial velocity and absolute luminosity has been considered in the communication by Kapteyn and Adams, to which reference has already been made. The observational material included here is much too limited to provide the basis for a discussion of this question. It may be noted, however, that the average radial velocity corrected for the solar motion of such absolutely faint stars as have been observed at Mount Wilson is exceptionally great. The average velocity of sixteen stars with absolute magnitudes below 8 on a scale for which the sun is 5.5 is 36 km. Eight have velocities exceeding 40 km. Since these stars are probably of small mass the evidence so far as it goes is in favor of Halm's hypothesis[5] of the equipartition of energy among the stars, their motions being a function of their masses.

[1] *British Association Report*, 1911.
[2] These PROCEEDINGS, 1, 14 (1915).
[3] *Lick Obs. Bull.*, No. 196.
[4] *Stellar Movements and the Structure of the Universe*, p. 157.
[5] *London, Mon. Not. R. Ast. Soc.*, 71, 634 (1911).

LOCALIZATION OF THE HEREDITARY MATERIAL IN THE GERM CELLS

By T. H. Morgan

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Presented to the Academy, May 17, 1915

It has come to be recognized that it must be more than a coincidence that in each animal and plant there are two representatives of each hereditary character (one derived from the mother and one derived from

the father), and that there are two of each kind of chromosome (one derived from the mother and one derived from the father). Moreover this relation becomes much more impressive when it is found that in the formation of the germ cells the representatives of the different characters separate from each other, and that the chromosomes separate also, so that each germ cell has but one set of factors and but one set of chromosomes. The comparison does not stop even here, and I propose to review briefly the further evidence that leads to the conclusion that the chromosomes are the bearers of the hereditary characters and that the known chromosomal behavior suffices as a mechanism to explain Mendel's law.

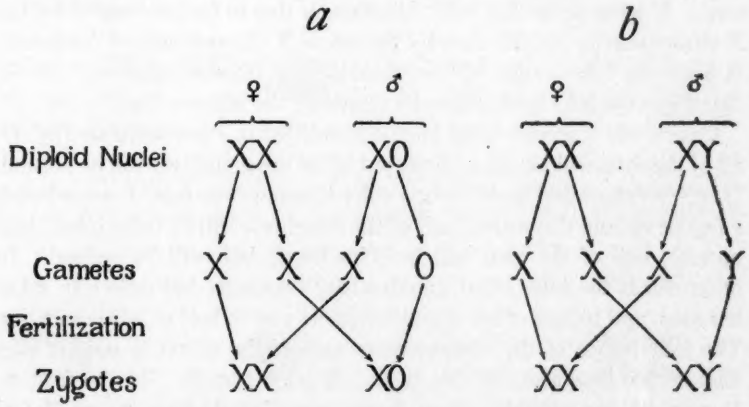

FIG. 1. DIAGRAMS OF SEX DETERMINATION; *A*, TYPE IN WHICH THE FEMALE IS XX AND THE MALE XO; *B*, TYPE IN WHICH THE FEMALE IS XX AND THE MALE XY.

Sex Linked Inheritance and the Sex Chromosomes. There is one exception to the rule of the dual nature of the chromosomes and at the same time an exception to the rule of the dual nature of the hereditary factors. In many species of animals it has been shown that while there are two sex chromosomes in the female, there is only one in the male (fig. 1 a). It is true in other animals often closely related to these, that the male may have one sex chromosome like the two in the female, and in addition a smaller Y chromosome, and in a few cases the Y may be as large as the X (fig. 1 b). In Drosophila there is an XY pair in the male, but the Y chromosome has been shown not to carry any factors that affect the characters due to factors carried by the X chromosomes. The influence of the Y is therefore nil, and the results are the same as though no Y were present.

The unequal distribution of the X chromosomes in the two sexes has furnished an opportunity to examine the theory that the chromosomes are the bearers of the hereditary factors, because it can be shown that the inheritance of a certain class of characters follows the known i stribution of the sex chromosomes. For example, color blindness in man is inherited in the following way (figs. 2 and 3). A color blind man married to a normal woman (fig. 2) produces normal sons and daughters. If two individuals that have had this origin should marry each other, the daughters will all be normal, half of the sons will be normal but half will be color blind. In other words the color blind grandfather has transmitted his defect to none of his granddaughters, but to half of his grandsons. If we assume that color blindness is due to factors carried by the X chromosomes—in this case by the single X chromosome of the grandfather—the inheritance of the chromosome (to the right) and of the defect (to the left) is the same, as shown by the scheme (fig. 2).

Conversely if a color blind female is married to a normal male (fig. 3), all of the sons will be color blind, but all of the daughters will be normal, If such a color blind male (or any color blind male in fact) is mated with a female having this origin, half of the daughters will be color blind, half normal; half of the sons will be color blind, half will be normal. In other words the color blind grandmother transmits her defect to all of her sons, and to half of her granddaughters and to half of her grandsons. The inheritance of the chromosomes and of the defect is strictly parallel in this case also, as seen in the diagram (fig. 3). In the fruit-fly Drosophila ampelophila there have been found over 40 sex-linked characters all of which are inherited according to the scheme given above. Moreover all possible combinations and tests that have been made—and there are many ways of testing such a conclusion—have given results that are entirely consistent with this scheme.

Linkage. If the hereditary characters are carried by the chromosomes we should expect that there would be great groups of characters that go together in inheritance, because while the number of the chromosomes is limited the number of the hereditary characters must be very great.

In Drosophila the number of pairs of chromosomes is only four, while over one hundred mutant characters have been studied. These characters fall into four great groups. The names of some of these characters that involve nearly all parts of the body, are given in the following list:

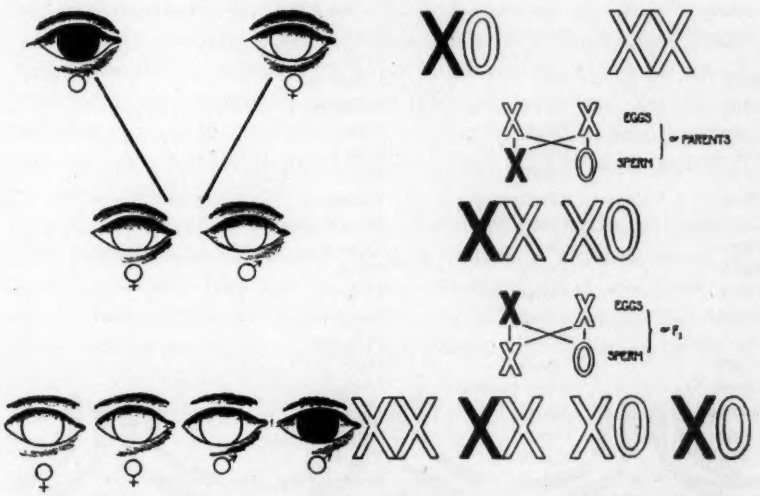

FIG. 2. DIAGRAM OF INHERITANCE OF COLOR BLINDNESS IN MAN. THE COLOR BLIND EYE AND THE X CHROMOSOME THAT CARRIES THE FACTOR FOR COLOR BLINDNESS ARE REPRESENTED IN SOLID BLACK. IN THIS DIAGRAM THE CHARACTER ENTERS THROUGH MALE.

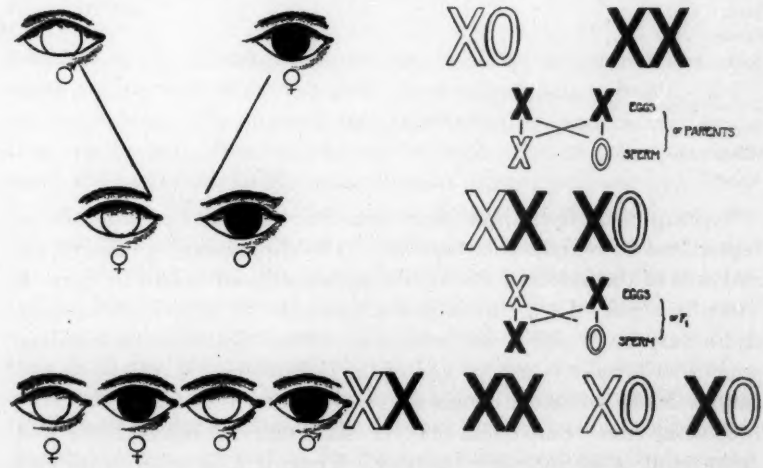

FIG. 3. AS IN FIG. 2, BUT HERE THE COLOR BLINDNESS ENTERS THROUGH THE FEMALE.

Group I	*Group II*	*Group III*	*Group IV*
Abnormal	Antlered	Band	Bent
Bar	Apterous	Beaded	Eyeless
Bifid	Arc	Cream III	
Bow	Balloon	Deformed	
Cherry	Black	Dwarf	
Chrome	Blistered	Ebony	
Cleft	Comma	Giant	
Club	Confluent	Kidney	
Depressed	Cream II	Low crossing over	
Dotted	Curved	Maroon	
Eosin	Dachs	Peach	
Facet	Extra vein.	Pink	
Forked	Fringed	Rough	
Furrowed	Jaunty	Safranin	
Fused	Limited	Sepia	
Green	Little crossover	Sooty	
Jaunty	Morula	Spineless	
Lemon	Olive	Spread	
Lethal's, 13	Plexus	Trident	
Miniature	Purple	Truncate intensifier	
Notch	Speck	Whitehead	
Reduplicated	Strap	White ocelli	
Ruby	Streak		
Rudimentary	Trefoil		
Sable	Truncate		
Shifted	Vestigial		
Short			
Skee			
Spoon			
Spot			
Tan			
Truncate intensifier			
Vermilion			
White			
Yellow			

A group consists of those characters that are inherited together, or rather *tend* to be inherited together. The chromosome groups of the male and of the female of Drosophila ampelophila are shown in figure 4. There is a pair of sex chromosomes, two pairs of large chromosomes, and a very small pair—four pairs in all. The coincidence between the number of hereditary groups and of the chromosomes is sufficiently evident to render comment unnecessary.

Crossing over. I have said that the characters that belong to the first group tend to be inherited together. Thus, if a fly with white eyes and yellow wings be crossed to the wild type, having red eyes and gray wings, the second generation of flies will be white yellow and gray red gray, *except* that once in a hundred times a fly will be found that is white

gray or else red yellow. In other words an exchange of characters has in some way taken place, but not oftener than once in a hundred times. We can see the exchange taking place in the reverse way if we cross a white eyed, gray winged fly to a red eyed, yellow winged fly. In the second generation the flies will now be white gray and red yellow, except that once in a hundred times a fly will appear that is white yellow or red gray.

Now if at the time when the chromosomes conjugate the two like chromosomes should exchange pieces, a consistent explanation of the results described above can be given. As a matter of fact there are several stages before, during and after conjugation of the chromosomes when interchange of pieces might take place. One of the phases is il-

FIG. 4. DIAGRAMS OF THE CHROMOSOMES OF DROSOPHILA AMPELOPHILA: TO THE LEFT, NORMAL FEMALE; IN THE MIDDLE, NORMAL MALE; TO THE RIGHT, NON-DISJUNCTIONAL FEMALE.

lustrated in the following diagram (fig. 5). Two homologous chromosomes are represented as each made up of a single line of beads. During conjugation, like chromosomes come together, and as they unite they can be seen often to twist around each other in consequence of which the parts come to line up on opposite sides of each other, as shown in the figure. When the chromosomes move apart the beads on one side are represented as passing to one pole, and those on the opposite side to the other pole; this means that during conjugation the pairs of factors, here represented by the beads, segregate independently of each other, and that the forces that draw the beads apart are often stronger than those that hold consecutive beads in their original line or series.

It may then, I think, be fairly claimed that the facts of crossing-over which must be due to interchange between homologous chromosomes, if chromosomes be admitted as the bearers of the hereditary factors, can be accounted for without departing from the known behavior of the chromosomes at the time of conjugation. On the other hand it should be clearly understood that the conclusion as to the way in which the

interchange takes place is not as yet on the same footing as are the other relations previously described.

Non-disjunction. In some experiments carried out by Mr. C. B. Bridges a race of flies was found that gave in 5% of cases exceptional results. There were 2½% of males that were like their father (patroclinous) when the expectation was that all of the males should be like their mother, in the sex-linked character in question; and there were 2½% of females that were like their mother (matroclinous) when the expectation was that they should all be like their father. From certain evidence that I can not now consider, Bridges was led to suspect that an additional sex chromosome was present in the mother that gave these

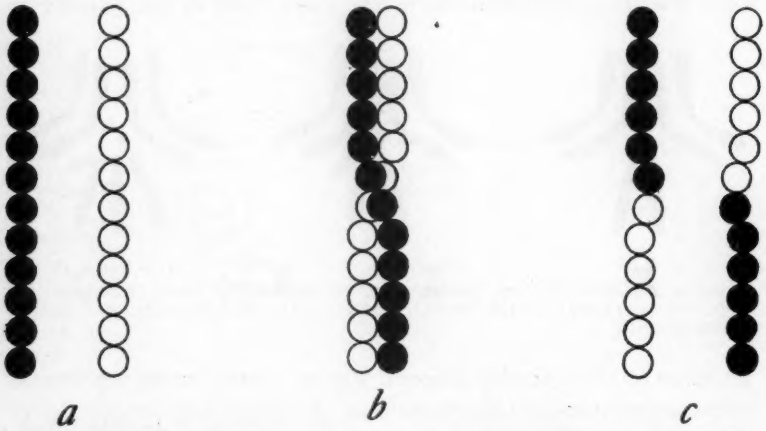

FIG. 5. DIAGRAM ILLUSTRATING CROSSING OVER OF THE MEMBERS OF A PAIR OF HOMOLOGOUS CHROMOSOMES REPRESENTED BY A SERIES OF BLACK AND A SERIES OF WHITE CIRCLES.

results; and from certain genetic evidence it was predicted that this chromosome could be none other than the Y chromosome. A cytological study of these females has demonstrated that the inference was correct.

These XXY females contain three elements that might be expected to come together during conjugation. The following situation then arises: X and X may conjugate and later pass to opposite poles (one going into the polar body), and Y would then be left to go to either pole. Two classes of eggs would result namely XY and X eggs. On the other hand X and Y might conjugate and pass to opposite poles (one going into the polar body) and the other X would then be left to go to either pole. The two classes of eggs that would result are XX

and XY. There would be expected then, in all, four classes of eggs from an XXY female. These might be fertilized by the female producing sperm of a male with the results shown in (1), (2), (3), (4) of figure 6; or by a male producing sperm with the results shown in the lower line of the same figure (5), (6), (7), (8). If the XXY female were a white eyed female (her two white bearing X's are represented by

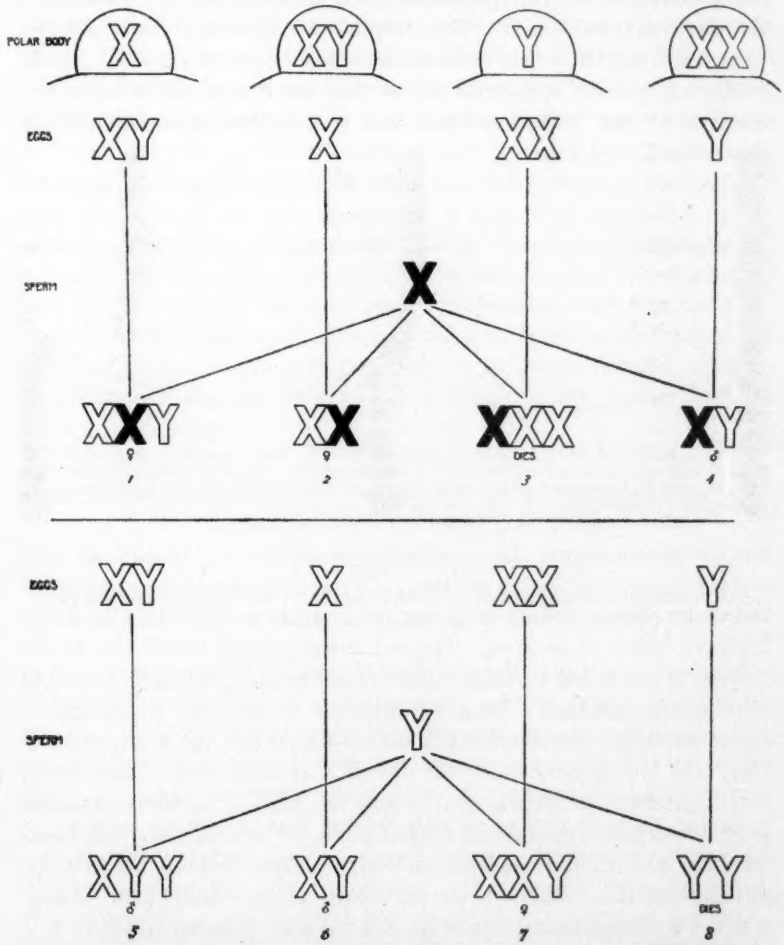

FIG. 6. DIAGRAM TO ILLUSTRATE THE PRODUCTION OF FOUR CLASSES OF EGGS OF A WHITE EYED FEMALE THROUGH NON-DISJUNCTION, AND THEIR FERTILIZATION BY AN X-BEARING SPERM, GIVING ZYGOTES 1, 2, 3, 4; AND BY Y-BEARING SPERM, GIVING ZYGOTES 5, 6, 7, 8.

the two open X's in the diagram) and if she were fertilized by a red eyed male (whose single X is represented by the black X in the diagram) the offspring will be expected to fall into the eight classes shown in the same diagram. The exceptional sons (4) arise when a Y bearing egg is fertilized by a female producing sperm (which here carries the factor for red). Thus a female producing sperm gives rise to a male, because maleness results from one X and not from anything else in the nature of the sperm that made him. The exceptional daughters (7) are the white eyed XXY females which arise when an XX egg is fertilized by a male producing sperm. Here again we see that femaleness is due to the occurrence of two X chromosomes, and it makes no difference whence these two X's have come.

There are many ways in which the hypothesis that non-disjunction is due to the presence of a Y chromosome, in the females that give these unique results, can be tested. For instance, the exceptional white

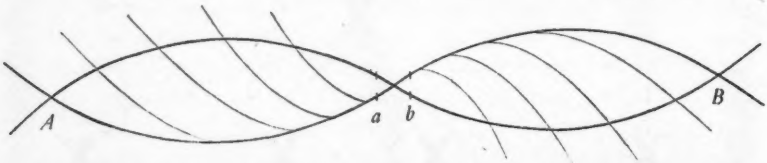

FIG. 7. DIAGRAM TO ILLUSTRATE INTERFERENCE: THE DISTANCE BETWEEN THE POINTS WHERE THE HEAVY LINES CROSS REPRESENTS THE AVERAGE DISTANCE BETWEEN TWO POINTS OF CROSSING OVER; THE LIGHTER LINES INDICATE THE LESS FREQUENT POINTS OF CROSSING OVER IN THE PROTECTED REGION.

females should repeat the non-disjunctional process. In fact, all such white females give non-disjunction. On the other hand the exceptional red males should neither give, nor transmit, non-disjunction, and this has been found to be true. The red females should be of two kinds; half (2) are expected to behave like normal females, and half (1) should show non-disjunction. These two kinds of females are found, and in equal numbers. There are two kinds of white males, XYY (5), and XY (6). The former should produce some XY spermatozoa. Such sperm would produce daughters, that would be XXY in composition, and from them non-disjunctional offspring are to be expected. This also has been tested and it has been found that half of the white males transmit through their daughters the peculiarity of non-disjunction. A diagram of a chromosome group of an XXY female is shown in figure 4.

Interference. If there is an average length of loop between crossing-over points, it would seem to follow that the region of the chromosomes on each side of the point where crossing over occurs would be pro-

tected, as it were, from another crossing over. This relation is illustrated in the diagram (fig. 7). If crossing over occurs at some point between two pairs of factors *a* and *b* indicated on the diagram, the next crossing over point of the chromosome would, if it occurred at all, lie at some distance away, rather than near by. This possibility can be tested in a case where several known points are present in a pair of chromosomes. If in such a case we determine how often crossing over occurs between *A* and *a* in general and then determine how often it occurs between *A* and *a* in those cases where it is known to have taken place between *a* and *b*, we find an enormous decrease in the number of times it occurs between *A* and *a* when at the same time it has occurred between *a* and *b*. In general one may say that crossing over at any level interferes with crossing over in the region of each side of that level.

Conclusions. The chromosomes not only furnish a mechanistic explanation of Mendelian heredity, but in the case of Non-disjunction and in the case of the point by point correspondence between the Linkage Groups and the chromosomes, furnish a *verifiable* explanation of the results. In the case of Crossing-over and of Interference the chromosomes give us the only objective explanation of the results that has been as yet offered.

RESEARCHES ON THE CHEMICAL AND MINERALOGICAL COMPOSITION OF METEORITES

By George P. Merrill

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Presented to the Academy, June 1, 1915

The primary motive of these investigations was to test the authenticity of numerous reported occurrences of certain minor constituents, such as antimony, arsenic, gold, lead, tin, tungsten, uranium, zinc, etc., and incidentally to formulate the analyses in such a way that the results might be made comparable with those of terrestrial rocks. Upwards of twenty meteorites were subjected to searching chemical analyses, with the particular end in view stated above. The results were in part confirmatory and in part contradictory. In none of the samples tested, either metallic or stony, could any traces be discovered of antimony, arsenic, gold, lead, tin, tungsten, uranium, or zinc. The presence in traces of platinum, palladium, iridium, ruthenium, and vanadium was, however, proved beyond apparent question, ruthenium being noted for the first time, and vanadium having previously been reported but once.

Attention was also directed toward the possible occurrence of barium, strontium, and zirconium, particularly in the feldspathic forms. Only negative evidence was obtained. This is not regarded as conclusive, but the investigation was hampered by a paucity of material.

Incidental to the work, a determined effort was made to ascertain to what of the minerals these minor constituents should be relegated. These results have thus far been only partially successful. It is evident that the platinum, palladium, and ruthenium, and perhaps the vanadium as well, are constituents of the metallic portions. The same is true of at least a part of the nickel and cobalt, although determinations made on the silicate portions, after most careful work to remove all of the native metal, still showed traces of both of these elements, indicating that they were also constituents of the pyroxenes or olivines as is often the case in terrestrial rocks.

In attempting to ascertain the source of the phosphorus in the silicate portion, unexpected results were developed, the presence being shown of a phosphate of lime differing from normal apatite in being optically biaxial and with a somewhat lower index of refraction. This, together with the apparent widespread occurrence of the mineral oldhamite, a sulphide of calcium, as indicated by aqueous solutions, has been made the subject of a special paper in the May number of these PROCEEDINGS.

A table comprising fifty-nine selected analyses of stony meteorites was given, twelve of which were made during the course of the investigations here being considered, others being taken from published descriptions by other workers. This table, too large for reproduction here, will appear with the extended report in some future publication. The average composition of the stony meteorites, as shown by fifty-three of the analyses, is given in column I of the following table. In column II is shown, for purposes of comparison, the average composition of the terrestrial lithosphere as given by F. W. Clarke, while in columns III and IV are the meteoric averages presented in previous papers by the author and Dr. O. C. Farrington respectively.

	I	II	III	IV
SiO_2	38.68	59.85	38.732	39.12
TiO_2	0.18[1]	0.73	.....	0.02
SnO_2	none			0.02
ZrO_2	none	0.03		
Al_2O_3	2.88	14.87	2.7333	2.62
Fe_2O_3		2.63	0.835	0.38
Cr_2O_3	0.47[2]	0.05		0.41
V_2O_3	trace	0.02		
Fe	11.98		11.536	11.46
Ni	1.15[3]		1.312	1.15
Co	0.07[4]			0.05

	I	II	III	IV
FeO	14.58	3.35	16.435	16.13
NiO	0.48[5]	0.03		0.21
CoO	0.06[6]			
CaO	2.42	4.81	1.758	2.31
BaO	none	0.10		
MgO	22.67	3.77	22.884	22.42
MnO	0.29[7]	0.09	0.556	0.18
SrO	none	0.04		
Na_2O	0.87[8]	3.29	0.943	0.81
K_2O	0.21[9]	3.02	0.328	0.20
Li_2O	trace	0.01		
H_2O(Ign.)	0.75[10]	2.05		0.20
P_2O_5	0.26[11]	0.25		0.03
S	1.80[12]	0.10	1.839	1.98
Cu	0.014[13]			
C	0.15[14]	0.03		0.06
Cl	0.08[15]	0.06		
F	?	0.10		
CO_2	?	0.70		
SO_3		0.02		
Ni, Mn, Cu, Sn				0.02
	100.04	100.00	100.00	99.82

[1] Average of 46 determinations
[2] Average of 42 determinations.
[3] Average of 50 determinations.
[4] Average of 41 determinations.
[5] Average of 19 determinations.
[6] Average of 6 determinations.
[7] Average of 33 determinations
[8] Average of 49 determinations.
[9] Average of 44 determinations.
[10] Average of 15 determinations.
[11] Average of 44 determinations.
[12] Average of 51 determinations.
[13] Average of 16 determinations
[14] Average of 8 determinations.
[15] Average of 5 determinations.

The detailed results of these investigations on the chemical and mineralogical constituents of meteorites begun in 1912 under a grant from the National Academy will be printed as a Memoir in the series of *Memoirs of the National Academy*.

ON THE REPRESENTATION OF ARBITRARY FUNCTIONS BY DEFINITE INTEGRALS

By Walter B. Ford

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Presented to the Academy, May 27, 1915

Let $f(x)$ be any function of the real variable x defined and with $|f(x)|$ integrable throughout the interval (a, b) and having limited total fluctuation in the neighborhood of the particular point $x = \alpha$ $(a < \alpha > b)$. Then, if $\varphi(n, x-\alpha)$ be a function of the parameter n and of $x-\alpha$ satisfying certain well known conditions the integral

$$I_n(\alpha) = \int_a^b f(x)\, \varphi(n, x-\alpha)\, dx \qquad (1)$$

will have the property that

$$\lim_{n=+\infty} I_n(\alpha) = \frac{f(\alpha-0)+f(\alpha+0)}{2}. \tag{2}$$

An important special instance of an integral (1) having the property (2) is presented in the study of the convergence of the Fourier series for $f(\alpha)$, in which case the sum of the first $n+1$ terms of the series can be put into the form (1) with $a=-\pi$, $b=\pi$ and $\varphi(n, x-\alpha)$ = a certain trigonometric expression.

While the conditions upon $\varphi(n, x-\alpha)$ that will insure (2) have been extensively studied, especially by Du Bois Reymond, Dini, Hobson and Lebesgue, relatively little appears to have been done in the actual determination of such functions, the sole desideratum being the determination of the conditions themselves. In this connection the present paper would point out a noteworthy class of possible functions φ with special emphasis upon the corresponding integrals (1) to which they give rise. Four theorems are established, the second being especially noteworthy in that it shows that to every convergent improper integral of the form

$$\int_0^\infty p(x)\,dx = k \neq 0,$$

wherein $p(x)$ satisfies very simple conditions, there can be made to correspond a certain integral (1) having the property (2).

The theorems are as follows:

Theorem I: Let $F(x)$ be any single valued function of the real variable x defined for all finite values of x and satisfying the following three conditions:

(*a*) $\lim_{x=+\infty} F(x)$ exists and $= k \neq 0$

(*b*) $F(-x) = -F(x)$.

(*c*) The derivative $F'(x)$ exists and is such that if we exclude the point $x=0$ by an arbitrarily small interval $(-\epsilon, \epsilon)$, $(\epsilon>0)$, we shall have for all remaining values of x, $|x\,F'(x)|<A_\epsilon$ = an assignable constant depending only on ϵ. Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b), we shall have for any special value α $(a<\alpha<b)$

$$\lim_{n=+\infty} \frac{1}{2k}\int_a^b f(x)\,\frac{d}{dx}F[n(x-\alpha)]\,dx = \frac{f(\alpha-0)+f(\alpha+0)}{2}$$

provided merely that $f(x)$ satisfies suitable conditions (analogous to

those under which the Fourier series for $f(\alpha)$ converges) in the neighborhood of the point $x=\alpha$.

Theorem II: Given any convergent improper integral of the form

$$\int_0^\infty p(x)\,dx = k \neq 0$$

wherein (a) the function $p(x)$ is even, i.e. $p(-x) = p(x)$, and (b) the expression $|x\,p(x)|$ for all values of x lying outside an arbitrarily small interval surrounding the origin remains less than a constant depending on the interval. Then if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b), we shall have for any special value α $(a<\alpha<b)$

$$\lim_{n=+\infty} \frac{n}{2k}\int_a^b f(x)\,p[n(x-\alpha)]\,dx = \frac{f(\alpha-0)+f(\alpha+0)}{2},$$

provided merely that $f(x)$ satisfies suitable conditions in the neighborhood of the point $x=\alpha$.

Theorem III: Let $F(x)$ be any single valued function of the real variable x which, when considered for positive (negative) values only of x satisfies the following three conditions:

(a)

$$\lim_{x=+\infty} F(x) \text{ exists and } = k \neq 0 \quad \left(\lim_{x=-\infty} F(x) \text{ exists and } = -k = 0\right)$$

(b) $F(0)=0$.

(c) The derivative $F'(x)$ exists and is such that if we exclude the point $x=0$ by an arbitrarily small interval $(-\epsilon, \epsilon)$, $(\epsilon>0)$, we shall have for all remaining positive (negative) values of x, $|x\,F'(x)|<A_\epsilon$ = an assignable constant depending only on ϵ.

Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b), we shall have for any special value α $(a<\alpha<b)$

$$\lim_{n=+\infty} \frac{1}{k}\int_\alpha^b f(x)\,\frac{d}{dx}F[n(x-\alpha)]\,dx = f(\alpha+0)$$

$$\left(\lim_{n=+\infty} \frac{1}{k}\int_a^\alpha f(x)\,\frac{d}{dx}F[n(x-\alpha)]\,dx = f(\alpha-0)\right),$$

provided merely that $f(x)$ satisfies suitable conditions at the right (left) of the point $x=\alpha$.

Theorem IV: Given any convergent improper integral of the form

$$\int_0^\infty p(x)\,dx = k \neq 0 \qquad \left(\int_{-\infty}^0 p(x)\,dx = k \neq 0\right)$$

wherein $|x\ p(x)|$ for all positive (negative) values of x lying outside an arbitrarily small interval to the right (left) of the point $x=0$ remains less than a constant depending on the interval.

Then, if $f(x)$ be an arbitrary function of the real variable x defined throughout the interval (a, b), we shall have for any special value α $(a<\alpha<b)$

$$\lim_{n=+\infty} \frac{n}{k}\int_\alpha^b f(x)\,p[n(x-\alpha)]\,dx = f(\alpha+0)$$
$$\lim_{n=+\infty} \frac{n}{k}\int_a^\alpha f(x)\,p[n(x-\alpha)]\,dx = f(\alpha-0),$$

provided merely that $f(x)$ satisfies suitable conditions in the neighborhood at the right (left) of the point.

The proof of Theorem I follows directly from the fact (see for example, Dini's *Serie di Fourier* (Pisa, 1880), pp. 119–121) that if $f(x)$ satisfies the indicated conditions there exists the general relation

$$\lim_{n=+\infty} \int_a^b f(x)\,\varphi(n, x-\alpha)\,dx = \frac{f(\alpha-0)+f(\alpha+0)}{2},$$

whenever $\varphi(n, t)$ is any function of the independent variables n and t satisfying the following three conditions, ϵ always denoting an arbitrarily small positive quantity:

(I) $\lim_{n=+\infty} \int_0^t \varphi(n,t)\,dt = \begin{cases} -\frac{1}{2} \text{ when } -\epsilon<t<0 \\ +\frac{1}{2} \text{ when } 0<t<\epsilon \end{cases}$

(II) $\left|\int_0^t \varphi(n,t)\,dt\right| < c_1$ when $-\epsilon<t<\epsilon$, c_1 being a constant (dependent only on ϵ)

(III) $|\varphi(n,t)| < c_2$ when $\begin{cases} a-\alpha<t<-\epsilon \text{ or} \\ o<t<b-\alpha \end{cases}$, c_2 being a constant (dependent only on ϵ)

Theorem II is a corollary of Theorem I.

Theorem III results from the fact (cf. Dini, l.c.) that if the conditions (I), (II), (III) above hold only for the positive (negative) values of t there specified, then, whenever $f(x)$ satisfies suitable conditions at the right (left) of the point $x=\alpha$, we may write

$$\lim_{n=+\infty} \int_{\alpha}^{b} f(x)\, \varphi(n, x-\alpha)\, dx = \tfrac{1}{2} f(\alpha+0)$$

$$\left(\lim_{n=+\infty} \int_{a}^{\alpha} f(x)\, \varphi(n, x-\alpha)\, dx = \tfrac{1}{2} f(\alpha-0) \right)$$

Theorem IV is a corollary of Theorem III.

While the forms of representation for an arbitrary function $f(x)$ afforded by the preceding theorems do not, strictly speaking, represent the function in terms of definite integrals, but rather in terms of the limits of such integrals as the parameter n increases to $+\infty$, it is to be observed that the first member of (2) may always be expressed as a convergent series, viz:

$$I_0(\alpha) + \sum_{n=0}^{\infty} [I_{n+1}(\alpha) - I_n(\alpha)]$$

and thus it appears that to every integral (2) obtained by any one of the preceding theorems there corresponds an actual representation of the arbitrary function in series of definite integrals.

THE LYMPHOCYTE AS A FACTOR IN NATURAL AND INDUCED RESISTANCE TO TRANSPLANTED CANCER

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Presented to the Academy, June 22, 1915

Histologically there is a striking resemblance between the series of phenomena which take place about a failing tissue graft in a host of a foreign species, and an homologous cancer graft in an animal with a natural or induced immunity to transplanted cancer. A constant finding in both cases is a local lymphoid reaction which appears early in the process, and lasts till the destruction of the tissue or cancer graft is complete. We have shown in previous communications that the lymphoid tissue is apparently the important factor in the destruction of a tissue graft in an animal of a foreign species. The facts which lead up to this conclusion are, that an organism like the chick embryo, which normally has no defensive agents against the cells of a foreign species, if supplied with adult lymphoid tissue becomes as resistant as the adult animal in this respect. Furthermore when the adult animal is deprived of the major portion of its lymphoid tissue by repeated small doses of X-ray, it loses the ability to destroy the cells of a foreign species and these will live and grow as well as they would in a native host. It

seemed of importance from these observations to study the lymphocytes in animals immune to transplanted cancers.

There are two types of this so-called immunity; the natural, possessed normally by a certain variable proportion of animals inoculated; and the induced variety, obtained by a previous treatment of the animal with an injection of homologous living tissues. We have studied the circulating lymphocytes in these two types of immunity and for comparison in susceptible mice with growing tumors.

In the mice with induced immunity the circulating lymphocytes show no change either in actual numbers or relative proportion to the other white cells during the ten days which must elapse between the immunizing tissue injection and the cancer inoculation. Within twenty-four hours after the cancer graft is introduced, however, there is a sharp rise in the number of these cells amounting to an average increase for the group of 100% above the former level. This increase continues with slight variations for something over 50 days, with a maximum average for the series of between 200 and 300% above the normal. The other white cells of the blood retain their normal level.

The natural immune animals, those which without treatment were able to overcome the cancer graft, showed a similar response on the part of the lymphocyte to that seen in the preceding group. The period of increase, however, is not evident for several days or a week after inoculation and the average maximum is not so high, being between 100 and 200% above the normal. Like the first group, there is slight, if any change in the actual numbers of the other white cells of the blood.

Animals in which the cancer graft resulted in a take, showed no such lymphoid response as did the immune animals. A composit curve of the white cells plotted for a number of such animals showed a slight tendency on the part of the lymphocyte to increase during the first two weeks, but this was followed by a gradual decline in number as the cancer increased in size. The polymorphonuclear cells during this period showed an actual increase in numbers.

In order to ascertain the importance of the lymphoid reaction in the immunity process, we have destroyed the major portion of the lymphoid tissue in mice having one of the two types of immunity. This was done by giving the animal several small exposures to X-ray, previously estimated to be sufficient for the purpose. In the induced immunity the X-ray was given between the time of the immunizing tissue injection and the cancer inoculation. This treatment resulted in the complete destruction of the immunity and the inoculated cancers grew more readily than in the normal animals.

For testing the importance of the lymphoid tissue in natural immunity we can only compare the percentage of takes in X-rayed and normal animals inoculated with the same cancer. This has been done with a variety of different cancers and a large series of animals. The average number of takes in the X-rayed animals was 94%, while in the untreated animals only 32% of those inoculated grew the cancer. This shows a very considerable destruction of the natural immunity accompanying a destruction of the lymphocytes.

To summarize, we have shown that a marked increase in the circulating lymphocytes occurs after cancer inoculation in mice with either a natural or induced immunity. When this lymphoid reaction is prevented by a previous destruction of the lymphoid tissue with X-ray the immune states are destroyed. Hence it would seem fair to conclude that the lymphocyte is a necessary factor in cancer immunity.

SOME THEOREMS CONNECTED WITH IRRATIONAL NUMBERS

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Presented to the Academy, May 27, 1915

As is well known to those who have investigated the fields of celestial mechanics, the series which arise there from the integration of the equations of motion involve factors of the form $(i-j\gamma)$ in the denominators of the coefficients, where i and j run over the entire series of positive integers and γ is a positive number which may be rational or irrational. Previous to the time when Poincaré had shown the existence and construction of periodic solutions (in which γ is always rational) it had been the custom for the astronomers to regard γ as irrational since with this hypothesis the factors $(i-j\gamma)$ never vanish and consequently non-periodic terms did not arise in the solutions. The presence of these factors in the denominators naturally led to very grave doubts as to the convergence of these series since there are infinitely many such factors which are smaller than any assigned limit, and the convergence has never been proved.

Considerations of this nature have led me recently to examine the convergence of simple types of power series in which this phenomenon occurs, and it has been found that the series $\Sigma\Sigma\, a_{ij}/(i-j\gamma)\, x_1^i x_2^j$ has precisely the same domain of convergence as the series $\Sigma\Sigma a_{ij}\, x_1^i x_2^j$, provided γ is a positive irrational number which satisfies a rather mild condition, which is stated below.

Further investigations in this field have resulted in the three following theorems:

Theorem I. If γ is any positive number, rational or irrational, and if p_n/n is a rational fraction such that $|p_n - n\gamma| \leqq \frac{1}{2}$, and if $A_n = \frac{1}{n}\sum_{\kappa=1}^{n}(p_\kappa - \kappa\gamma)$ is the arithmetic mean of the first n of the quantities $(p_\kappa - \kappa\gamma)$, signs considered, then the limit of A_n, as n increases without limit, is zero.

If $\gamma = p/q$ is rational with an even denominator q then for certain values of κ there are two integers p_κ which differ by unity such that $|p_\kappa - \kappa\gamma| = \frac{1}{2}$; for one, the value is $+\frac{1}{2}$ and for the other it is $-\frac{1}{2}$. It is supposed that such terms are taken alternately $+\frac{1}{2}$ and $-\frac{1}{2}$.

Theorem II. If γ is a positive number, and if p_n/n is a rational fraction such that $|p_n - n\gamma| \leqq \frac{1}{2}$, and if $A_n = \frac{1}{n}\sum_{\kappa=1}^{n}|p_\kappa - \kappa\gamma|$ is the arithmetic mean of the first n of the quantities $|p_\kappa - \kappa\gamma|$, signs discarded, then the limit of A_n, as n increases without limit, is $\frac{1}{4}$ if γ is irrational or rational with an even denominator; but if γ is rational with an odd denominator, $\gamma = p/q$, then the limit of A_n is $(q^2 - 1)/4q^2$.

Theorem III. If γ is a positive number, and if p_n/n is a rational fraction such that $|p_n - n\gamma| \leqq \frac{1}{2}$, and if $G_n = \left(\prod_{\kappa=1}^{n}|p_\kappa - \kappa\gamma|\right)^{\frac{1}{n}}$ is the geometric mean of the first n of the quantities $|p_\kappa - \kappa\gamma|$, then the limit of G_n, as n increases without limit, is zero if γ is rational, and is equal to $1/(2e)$ where $e = 2.71828\ldots$ is the naperian base, if γ is an irrational number which satisfies the condition

$$a_{n+1} \leqq M\, q_n\,(q_{n+1}) \ldots (q_{n+s}),$$

where γ, expressed as a simple continued fraction, is

$$\gamma = a_1 + \frac{1}{a_2+}\,\frac{1}{a_3+}\cdots,$$

q_n is the denominator of the n^{th} principal convergent, and s any assigned positive integer independent of n. If γ is an irrational number which does not satisfy this condition then G_n for large values of n oscillates between zero and $1/(2e)$.